# MODELING OF THE SENSITIVITY OF FAN-BEAM COLLIMATION IN SPECT IMAGING

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## Abstract

An essential feature of SPECT imaging is collimation of gamma photons, allowing detection of only those photons propagating in the appropriate direction. Parallel beam collimators are widely used in different applications, because reconstruction is straightforward. Fan-beam collimators focus on a focal line parallel to the axis of rotation. For organs centered within the field of view this enhances the sensitivity. We propose a new, more sophisticated model for calculating the sensitivity of fan-beam collimation throughout the whole field of view of an Anger gamma camera and compare it with a previously published model. Projection measurements of a uniform flood source near the face of a fan-beam collimator demonstrate that this new model is more accurate.

## 1 INTRODUCTION

The purpose of radionuclide imaging is to obtain an image of the distribution of a radioactive labeled substance within the body, after it has been administered to a patient. This is accomplished by recording the radioactivity emissions with the external radiation detector(s) of an Anger scintillation camera placed in the vicinity of the patient. A collimator is used to focus incoming rays at various projections. If there are enough projections obtained from different angles around the patient, a three dimensional image of the source distribution can be reconstructed. This is called a SPECT image (Single-Photon Emission Computed Tomography). The collimator types in SPECT imaging can be classified into parallel and focusing collimators. Parallel hole collimators represent a good compromise between resolution and sensitivity and are very versatile. Moverover, reconstruction is straightforward. Therefore, this type of collimation is widely used in clinical routine. On the other hand, focusing or convergent beam collimation enhances sensitivity and is useful when imaging smaller organs such as the brain and heart. fan-beam collimators are a special type of converging collimators, with the holes focusing towards a so-called focal line parallel to the axis of rotation of the camera. This study investigates the sensitivity of fanbeam collimation more extensively and proposes a new formalism for calculating the sensitivity throughout the field of view. We compared this model with the previously published model [1, 2] by means of sensitivity data, provided

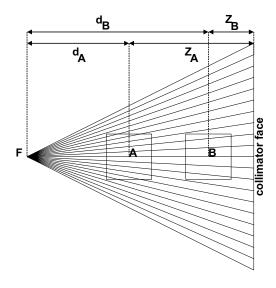


Figure 1: Intuitive approach to the sensitivity of a fan-beam collimator with focal point F. The sensitivity in A and B is proportional to respectively  $\frac{1}{dA}$  and  $\frac{1}{dB}$ 

by the manufacturer and by projection measurements of a uniform flood source.

## 2 THEORY

# 2.1 Ideal collimation

Consider a schematic representation of a fan-beam collimator with focal point F, where each line represents a direction for which radiation can be detected (see Figure 1). The collimator sensitivity in a point in the FOV (Field of View) is proportional to the density of lines going through a surface element in that point. Consider two points A and B with a different distance to the face of the collimator. The number of lines going through a surface segment in A is higher than the ones going through a surface segment of identical size in B. Therefore the collimator sensitivity is higher in A than in B. Similar to the radiation intensity of a point source in one dimension, which is inversely proportional to the distance between the point of measurement and the source, the density of lines and therefore the collimator sensitivity in a point in the FOV is inversely proportional to the distance between that point and the focal point

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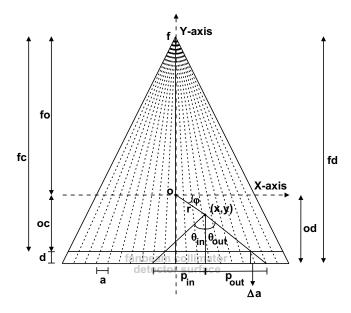


Figure 2: Schematic drawing of the coordinate system and symbols used in the calculation of the sensitivity maps

of the collimator.

In the case of a parallel beam collimator and following the above reasoning, one can easily understand that the line density through a surface segment in a arbitrary point of the FOV is constant and independent of the distance between that point and the collimator face. Therefore the collimator sensitivity of a parallel beam collimator is constant throughout the FOV. According to this approach, the collimator sensitivity of a fan-beam collimator in a point (x,y) (see Figure 2) in the FOV is given by:

$$S(x,y) = S_0 \frac{fc}{\sqrt{(fc - (oc + y))^2 + x^2}},$$
 (1)  
 
$$0 < \sqrt{x^2 + y^2} < oc.$$

 $S_0$  is the geometric sensitivity near the collimator face (and equivalent with the sensitivity for parallel beam geometry  $(fc \to \infty)$ ), fc the focal length of the fan-beam collimator and oc the radius or the distance between center of rotation and collimator face.

An approximation of this expression can be found in the literature [1, 2] and is given by:

$$S = S_0 \frac{fc}{fc - z}, 0 < z < fc.$$
 (2)

with z the source-detector distance.

Using the same coordinate system as (1), (2) corresponds with the following expression:

$$S(x,y) = S_0 \frac{fc}{(fc - (oc + y))},$$

$$0 < \sqrt{x^2 + y^2} < oc.$$
(3)

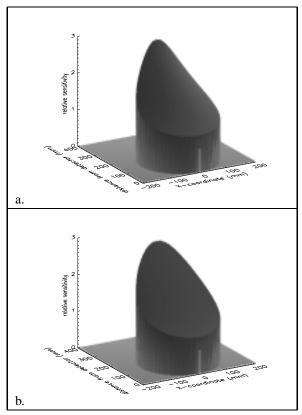


Figure 3: Geometric sensitivity throughout the FOV of a fan-beam collimator ( $fc=397~{\rm mm},\,oc=132~{\rm mm}$ ) calculated according to 2(a) and 3(b)

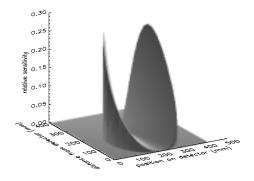


Figure 4: Difference image between Figure 3b and Figure 3a, shown as a surface

Comparing (3) with (2), the independence of x of the former can be noticed.

To illustrate the importance of this approximation in (3), the sensitivity is calculated throughout the FOV for a fanbeam collimator with focal distance fc=397mm and radius oc=132mm (fan-beam and clinical setting used for routine brain SPECT imaging) according to (2) (see Figure 3a) and according to (3) (see Figure 3b), together with

the difference image (see Figure 4). As could be expected, the fan-beam collimator sensitivity according to (3) is systematically overestimated, except for points on or near the y-axis.

# 2.2 New formalism

If yd = y + oc + d, xd = |x| and fd = fc + d, then

$$\theta_{out} = \arctan(\frac{p_{out}}{vd}) \tag{4}$$

and

$$\theta_{out} = \arctan(\frac{a + \Delta a}{d}) \tag{5}$$

with  $\Delta a = \frac{(p_{out} + xd - a) \times d}{fd}$ . From (4) and (5), one gets the following expression for  $p_{out}$ :

$$p_{out} = \left(\frac{a}{d} + \frac{xd}{fd} - \frac{a}{fd}\right) \times \frac{fd \times yd}{fd - yd} \tag{6}$$

The same goes for  $\theta_{in}$ 

$$\theta_{in} = \arctan(\frac{p_{in}}{yd}) \tag{7}$$

Whether  $p_{in}$  is either smaller or bigger the xd gives two expressions for  $\theta_{in}$ , that is:

$$\theta_{in} = \arctan(\frac{a + \Delta a}{d}) \tag{8}$$

with  $\Delta a = \frac{(p_{in} - xd - a) \times d}{fd}$  if  $p_{in} \geq xd$ , and

$$\theta_{in} = \arctan(\frac{a - \Delta a}{d}) \tag{9}$$

with  $\Delta a = \frac{(-p_{in} + xd + a) \times d}{fd}$  if  $p_{in} < xd$ .

(7) and (8) as well as (7) and (9) yield the same expression for  $p_{in}$ :

$$p_{in} = \left(\frac{a}{d} - \frac{xd}{fd} - \frac{a}{fd}\right) \times \frac{fd \times yd}{fd - yd} \tag{10}$$

Substitution of (10) in (7) and (6) in (4) and summing up  $\theta_{out}$  and  $\theta_{in}$  gives for a point (x, y) in the FOV the angle under which radiation can be detected:

$$\theta(x,y) = \theta_{out}(x,y) + \theta_{in}(x,y)$$

$$= \arctan(\left(\frac{fd}{fd - (y + oc + d)}\right)$$

$$\times \left(\frac{a}{d} + \frac{|x|}{fd} - \frac{a}{fd}\right)$$

$$+ \arctan(\left(\frac{fd}{fd - (y + oc + d)}\right)$$

$$\times \left(\frac{a}{d} - \frac{|x|}{fd} - \frac{a}{fd}\right)$$

The sensitivity according to (11) is calculated throughout the FOV for a fan-beam collimator with focal distance fc = 397mm and radius oc = 132mm (see Figure 5).

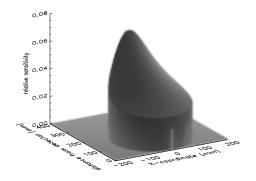


Figure 5: Geometric sensitivity throughout the FOV of a fan-beam collimator (fc = 397 mm, oc = 132 mm) calculated according to (11)

### 3 RESULTS

We have compared the three approaches, (1), (3) and (11) by means of sensitivity data, provided by the manufacturer and measured at the center of the collimator and at different distances from the the collimator face. Being measured at the collimator center means that (1) and (2) will yield the same sensitivity values. The results are shown in Figure 6, where all curves were scaled to the sensitivity measured at distance zero from the collimator face. All models show good correspondence with the measured values.

## COLLIMATOR SENSITIVITY

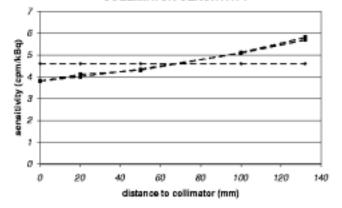


Figure 6: Sensitivity values for fan-beam at the collimator center ( $-\blacksquare$ -:values of manufacturer,  $-\blacktriangle$ -:values according to (1) and 3,  $-\bullet$ -:values according to (11)) and  $-\blacklozenge$ -:parallel beam (values of manufacturer) collimation

In order to get an idea of the correspondence off-center, sensitivity profiles near the collimator face were calculated according to the (1), (3) and (2). These are plotted in Figure 7 together with the measurement data of a uniform flood source, positioned at the face of the collimator. Once

again all curves were scaled to the maximum of the measured profiles. These data clearly show a better correspondence of the newly proposed sensitivity model with the

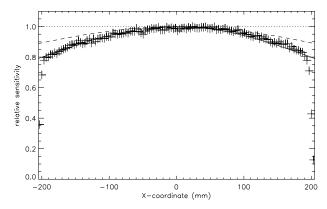


Figure 7: sensitivity profiles near the collimator face according to the (11) (solid line),(3) (dashed line) and (1) (dotted line), and measured profile of a uniform flood source at the face of the collimator (+)

## 4 CONCLUSION AND DISCUSSION

Phantom measurements have proved that the sensitivity model for fan-beam collimation, proposed here is accurate than the simplified models previously used. Indeed, the sensitivity at the center of a fan-beam collimator is higher, relative to the off-center sensitivity. For organs centered within the field of view this enhanced sensitivity compensates for the photon attenuation that is greatest in deep structures. Therefore, in the case of an accurate modeling of the attenuation effect during iterative reconstruction, an incorrect sensitivity model may bias in the quantification of SPECT images, acquired with a fan-beam collimator.

Another characteristic of each model is the average sensitivity over the different angles during rotation in a point of the FOV at distance r from the center of rotation. Therefore, one has to integrate (1), (3) and (11) over the angle range of rotation while dividing by this angle range. This gives the following expressions for an angle range of rotation of  $2\pi$ :

$$\overline{S(r)} = \frac{S_0}{2\pi} \int_0^{2\pi} \frac{fc \,\mathrm{d}\varphi}{\sqrt{(fc - (oc + r\cos(\varphi))^2 + (r\sin(\varphi))^2}},$$
(12)

$$\overline{S(r)} = \frac{S_0}{2\pi} \int_0^{2\pi} \frac{f c \,\mathrm{d}\varphi}{\sqrt{(f c - (oc + r \sin(\varphi)))}} \,, \tag{13}$$

$$\overline{S(r)} = \frac{1}{2\pi} \int_0^{2\pi} \frac{\theta(r\cos(\varphi), r\sin(\varphi)) \,d\varphi}{2\pi} , \qquad (14)$$

$$0 < r < oc$$

The sensitivity profiles throughout the FOV, obtained for a fan-beam collimator with fc=397mm and oc=132mm

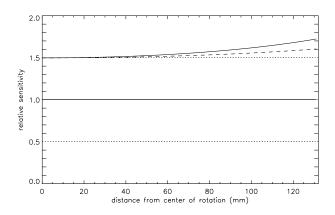


Figure 8: Average radial sensitivity for fan-beam according to (12) (dashed line) and (13) (solid line) and parallel beam (solid line with constant value 1)

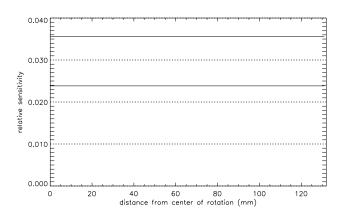


Figure 9: Average radial sensitivity for fan-beam and parallel beam (lowest value) according to (14)

are shown in Figure 8 and Figure 9. This average sensitivity is interesting for SPECT imaging because it gives an idea about the final reconstructed sensitivity in the SPECT image. One can conclude that the average sensitivity based on the previous model is slightly non-uniform throughout the FOV, whereas with the new model a uniform average sensitivity is obtained. This means that no post-reconstruction correction for non-uniform sensitivity is necessary.

### 5 REFERENCES

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